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Traffic Matrix Estimations directions and trends

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Just a few words on the presenter:

- my name is Davide Cherubini and I’m part of the Tiscali International Network’s Research and Development division

- In R&D we work on several fields, including **Network optimization models**
- **origin-destination traffic matrices determination**
- linear (and nonlinear) programming models for traffic optimization (fluxes layout/metrics optimization)
- topological strategies for networks evolution

- We assist our colleagues in the Engineering and in the Operations Depts.

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Who am I?

Problem definition

- Network Engineers have been building very large scale networks at their best, trying to understand beforehand (planning) and react just in time to the network events

- Prerequisite to effectively (or even optimally) re-arrange the traffic across the network, is understanding what is the relative size of each Origin-Destination flow. This problem is called the Traffic Matrix determination or estimation

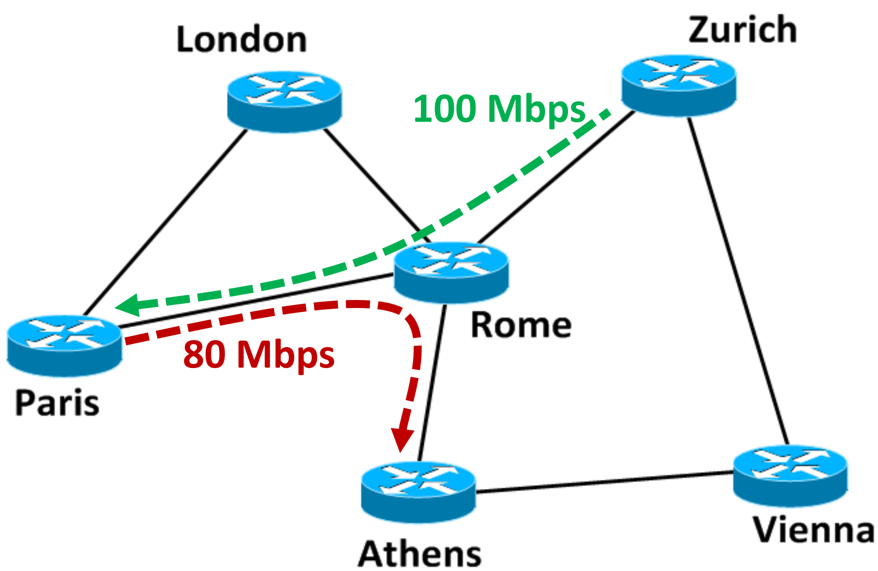
- In the last few years several methods have been presented because direct measurement are way too computationally intensive thus unfeasible

Why would we want to know the Traffic Matrix?

- **Traffic Engineering**
  - Optimal traffic distribution
  - Best routing path selection
  - Reasonable dimensioning
  - Provisioning
  - Survivability of the network
- **Planning**
- **Anomaly Detection**

«*The knowledge of a correct TM is highly valuable***»**

… no, not really!



Origins \ Destinations	Paris	Athens	Zurich
Paris	0	80	…
Athens	…	0	…
Zurich	100	…	0

- Each element of the TM reflects how much traffic is flowing from a source node to a destination node.
- It may represent either an **average** value or an **instant** value or a **peak** value
- The whole set of Origin-Destination flows is called **Traffic Matrix** (TM)

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What's a TM?

Mathematical formulation of the problem

Let us define:

$\mathcal{G}(N,E)$	the undirected graph composed of N nodes and E edges
$f_{i,j}$	the flux between nodes i and j (vector representation of the TM)
$c_{l,m}$	the capacity between nodes l and m
$\mathbf{M}$	the $2\|E\| \times (\|N\| \cdot \|N-1\|)$ matrix describing the routing

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Now some math! ;)

Algebraically speaking, the problem can be simply stated as follow

$$\begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_E \end{bmatrix} = \begin{bmatrix} m_{11} & m_{11} & \cdots & \cdots \\ m_{21} & m_{22} & \cdots & \cdots \\ \vdots & \vdots & \ddots & \ddots \end{bmatrix} \cdot \begin{bmatrix} f_{AB} \\ f_{AC} \\ f_{AD} \\ \vdots \\ f_{BA} \\ f_{BC} \\ f_{BD} \\ \vdots \end{bmatrix}$$

Unknown flows (TM)

SNMP link counts

Routing Matrix

- Or in its compact form as $\underline{c} = \mathbf{M} \cdot \underline{f}$

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Just a little bit more!

- The system is highly **under-determined** (# columns >> # rows)
- It is **ill-conditioned**
- If the rank of $\mathbf{M}$ is full, then the number of **possible solutions** is equal to

$$\infty^{(|N|\cdot|N-1|)-2|E|}$$

- the rank of $\mathbf{M}$ is full ***iff*** **all** the arcs are used by the internal routing protocol
- The solutions provide precious indications on
  - boundary of admittable capacity
  - pathological nodes
  - network stability

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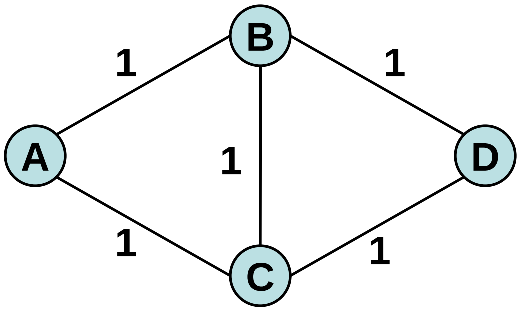
Some consideration

An example (1/3)

The simple graph shown below has 4 nodes, 5 bidirectional edges, and unitary metric values.



	f_{AB}	f_{AC}	f_{AD}	f_{BA}	f_{BC}	f_{BD}	f_{CA}	f_{CB}	f_{CD}	f_{DA}	f_{DB}	f_{DC}
A→B	1	0	0.5	0	0	0	0	0	0	0	0	0
A←B	0	0	0	1	0	0	0	0	0	0.5	0	0
A→C	0	1	0.5	0	0	0	0	0	0	0	0	0
A←C	0	0	0	0	0	0	1	0	0	0.5	0	0
B→C	0	0	0	0	1	0	0	0	0	0	0	0
B←C	0	0	0	0	0	0	0	1	0	0	0	0
B→D	0	0	0.5	0	0	1	0	0	0	0	0	0
B←D	0	0	0	0	0	0	0	0	0	0.5	1	0
C→D	0	0	0.5	0	0	0	0	0	1	0	0	0
C←D	0	0	0	0	0	0	0	0	0	0.5	0	1

- 12 columns (O-D flows)
- 10 rows (links)
- <u>The linear system has ∞^2 solutions!</u>

In this example we consider equal cost multiple paths (ECMP)

- We can face the problem using several methods finding, in turn, one among the infinite solutions

- The problem was tested with the following solvers:
  - General backslash operator (linear systems solver)
  - Linear Programming Solver (Interior Point)
  - Non Negative Least-Square Method
  - Conjugate-Gradient Method
  - Linear Least-Square Method with constraints

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An example (2/3)

The vector representation of the synthetic OD-Matrix is:

$$\underline{f_{orig}} = \left[\begin{array}{cccccccccccc} 10 & 10 & 10 & 10 & 10 & 10 & 10 & 10 & 10 & 10 & 10 & 10 \end{array}\right]$$

that, subjected to the Routing Matrix $\mathbf{M}$, determines the arc capacity vector:

$$\underline{c} = \left[\begin{array}{cccccccccc} 15 & 15 & 15 & 15 & 15 & 15 & 15 & 15 & 15 & 15 \end{array}\right]$$

Using the five aforementioned methods the obtained results are:

$$\underline{f_{solve}} = \left[\begin{array}{cccccccccccc} 15 & 15 & 0 & 15 & 10 & 15 & 15 & 10 & 15 & 0 & 15 & 15 \end{array}\right]$$

$$\underline{f_{lin}} = \left[\begin{array}{cccccccccccc} 9.3 & 9.3 & 11.2 & 9.3 & 10 & 9.3 & 9.3 & 10 & 9.3 & 11.2 & 9.3 & 9.3 \end{array}\right]$$

$$\underline{f_{nnls}} = \left[\begin{array}{cccccccccccc} 0 & 0 & 30 & 0 & 10 & 0 & 0 & 10 & 0 & 30 & 0 & 0 \end{array}\right]$$

$$\underline{f_{lsqr}} = \left[\begin{array}{cccccccccccc} 1 & 1 & 28 & 14.5 & 10 & 1 & 14.5 & 10 & 1 & 1 & 14.5 & 14.5 \end{array}\right]$$

$$\underline{f_{ssqr}} = \left[\begin{array}{cccccccccccc} 7.5 & 7.5 & 15 & 7.5 & 10 & 7.5 & 7.5 & 10 & 7.5 & 15 & 7.5 & 7.5 \end{array}\right]$$

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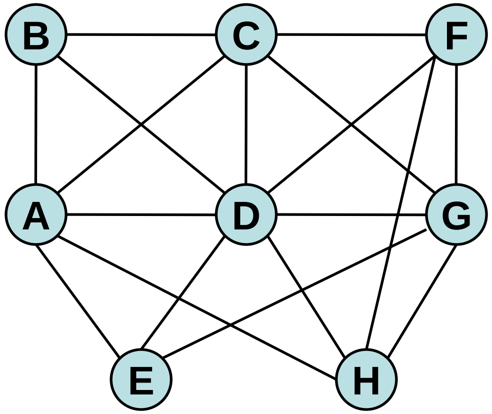
An example (3/3)

Adding links may not be your best friend approach:



Nodes: 8
Arcs: 36
Flows: 8x7=56

- The analysis of this network helped us to understand the relation between adding arcs to the network and the stability of the flow solutions found
- It seems that on a planar and simple network, traffic optimization is easy to perform. When the number of arcs increases, the problem of optimization becomes more difficult and the maximal capacity decreases, to increase again when the network is transformed into its clique

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Solutions?

Principal Component Analysis (PCA)

- The PCA reduces the dimensionality of the problem identifying the components having maximum energy (or variability)

- It has been demonstrated that a set of OD flows ***measured*** over large time scale can be represented by the sum of a small number of *eigenflows* (<10)

- The problem of eigenflows evaluation is **well-posed** then a solution exists and is **unique**!

Problem: the set of flow measurements is needed!

- PCA techniques are used in large number of applications such as databases, regression analysis, cluster analysis, intrusion analysis, and many others

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Component Analysis (1/2)

Independent Component Analysis (ICA)

- ICA can be seen as an extension of the PCA and identifies the *independent* components that are *non-gaussian* and *mutually independent.*

- The method is widely used in Audio Signal Processing, medical diagnosis (e.g. MEG*), and digital image correction

Curvilinear Component Analysis (CCA)

- The CCA/CDA (Curvilinear Distance Analysis) is an extension of the PCA for non-linear domains

* Magnetoencephalography

Component Analysis (2/2)

Is our graph a good graph?

- How simple or possible would be to remap the OD-Flows on the network.

- The eigenvalues and the eigenvectors of the Adjacency (or the Laplacian) matrix show interesting properties

In example it is possible to easily evaluate:

✓ the *isomorphism* of different graphs

✓ how *"well connected"* the graph is (that is represented by the magnitude of the second smallest eigenvalue)

- Methods available in literature need direct flows measurement in order to assist in strategy and tactics definition
- The best approach would be augmenting the LP Model with real traffic data, but we understand that flows sampling is a difficult and burdensome task

To sum up:

- ✓ *pure analytic methods are advantageous but subject to errors that cannot be computed*
- ✓ *Hybrid methods (with direct methods for flows determination) are better but not enough*
- ✓ *New techniques and research trends look promising*

«We are in any case providing estimated TM data to total-survivability models to evaluate robustness of the arc-failing predictions and protections»

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Conclusions

Any Question? (now…)

(…or later)

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  - R&D Labs
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Thanks

Backup slides

1st Generation Methods

- Introduce additional constraints with simple models for OD pairs (e.g. Poisson, Gaussian distribution)
- Highly dependent upon an initial prior estimate of the TM
- Neither spatial nor temporal correlation

2nd Generation Methods

- Extra SNMP measurement data (e.g. from access and peer links). Gravity models (Tomogravity)
- Explicitly change the routing, by changing the link metrics. This method increases the rank of the Routing Matrix by adding new rows
- Dynamic model intended to capture the temporal evolution of OD flows by mean of a Fourier expansion plus a noise model. It further increase the rank of the Routing Matrix

Looking back… (1/2)

3rd Generation Methods

- Fanout
  - Defined as "the vector capturing the fraction of incoming traffic that each node forwards to each egress node inside the network" [Crovella, et. al. SIGMETRICS '05]
  - Purely based on traffic measurements. Neither Routing Matrix nor inference is used

- Principal Component Analysis (PCA)
- Kalman Filtering
  - Directly derived from Dynamic System Linear Theory
  - Spatio-temporal correlations within a single equation
  - The Kalman filter is the best linear minimum variance estimator in the case of zero mean Gaussian noise

In order to correctly calibrate 3G models, at least 24 hours of traffic measurement is needed!!

Looking back… (2/2)

- If we deal with simple examples, in which the number of nodes and arcs is relatively small, we can definitely employ a direct strategy to find the optimal traffic flows layout.

- When the problem becomes large, it in necessary to find appropriate numerical strategies to overcome either in space or time complexity.

- Various techniques can be employed, but we found that in various cases, it is a good starting point understanding how well behaving is your network graph and IGP weights by factoring the arc-flows matrix via the Singular Value Decomposition (SVD).

- The SVD provides a lot of useful information on the problem. In example it provides an hint on how sensible the problem is on error on the data (conditioning), and it is a starting point towards finding the Moore-Penrose pseudo-inverse.

- This matrix provides a solution to the problem in the least-square sense.

Large Problems Tactics

- From SVD theory we know that any real matrix $\mathbf{M}$ can be factorized in the product of two unitary matrices $\mathbf{U}$ and $\mathbf{V}$ and a diagonal matrix Σ

$$\mathbf{M} = \mathbf{U} \cdot \Sigma \cdot \mathbf{V}^T$$

- This decomposition is important as the Moore-Penrose Pseudoinverse can be calculated as follow

$$\mathbf{M}^+ = \mathbf{V} \cdot \Sigma^{*T} \cdot \mathbf{U}^T$$

- Where Σ^{*T} is the matrix constructed replacing all non-zero elements with their reciprocal (its “inverse”)

- The pseudoinverse has some nice properties of the inverse. In fact it can be multiplied to the Capacities Vector provides the solution of the linear system $\underline{c} = \mathbf{M} \cdot \underline{f}$ that minimizes the Euclidean norm $||\mathbf{M} \cdot \underline{f} - \underline{c}||^2$

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From SVD to Pinv